Causal graphical models

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The theory of graphical causal models

A theory for measurement of average causal effect (ACE)

Basic assumptions:

- **A causal model has to be a special kind of chain graph model referred to as a "directed acyclic graph" (DAG).**
- **There must be a data generating DAG mechanism behind all legitimate data.**
- **Measurement of ACE is unconfounded in randomized experiments**

Raw calculation of ACE in observational studies may, but does not have to be confounded.

The theory provides

- **rules to identify situations where the raw ACE from observational studies is not confounded**
- **procedure for estimation of adjusted ACE from longitudinal observational studies**
- **(procedures for estimation of causal order)**

Some references:

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Eerola, M. (1993). ***On Predictive Causality in the statistical analysis of a series of events.*** University of Oulu: dept. of Appl. Math & Stat.

McKim, V.R. & Turner, P. (ed.s)(1998) ***Causality in Crisis.***

Richardson, T.S. (1998). Chain Graphs and Symmetric Associations. In Jordan, M.: ***Learning in Graphical Models***. pp. 231-260, Dordrecht: Kluwer Academic Publishers.

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Parner, J. (1999). ***Causal reasoning and time-dependent confounding.*** KU: Dept. of Biostat.

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"In the last decade, small groups of statisticians, computer scientists, and philosophers have developed a theory about how to represent causal relations ... . From those representations there follow accounts of how ... causal relations can be reliable learned, at least by computers." **(Glymour, 2001).**

"I have been doing applied statistics now for many years and have learned ... a couple of general principles from this experience ...:

a) The law of two numbers. If you get two different numbers that are supposed to be the same, at least one of them is wrong.

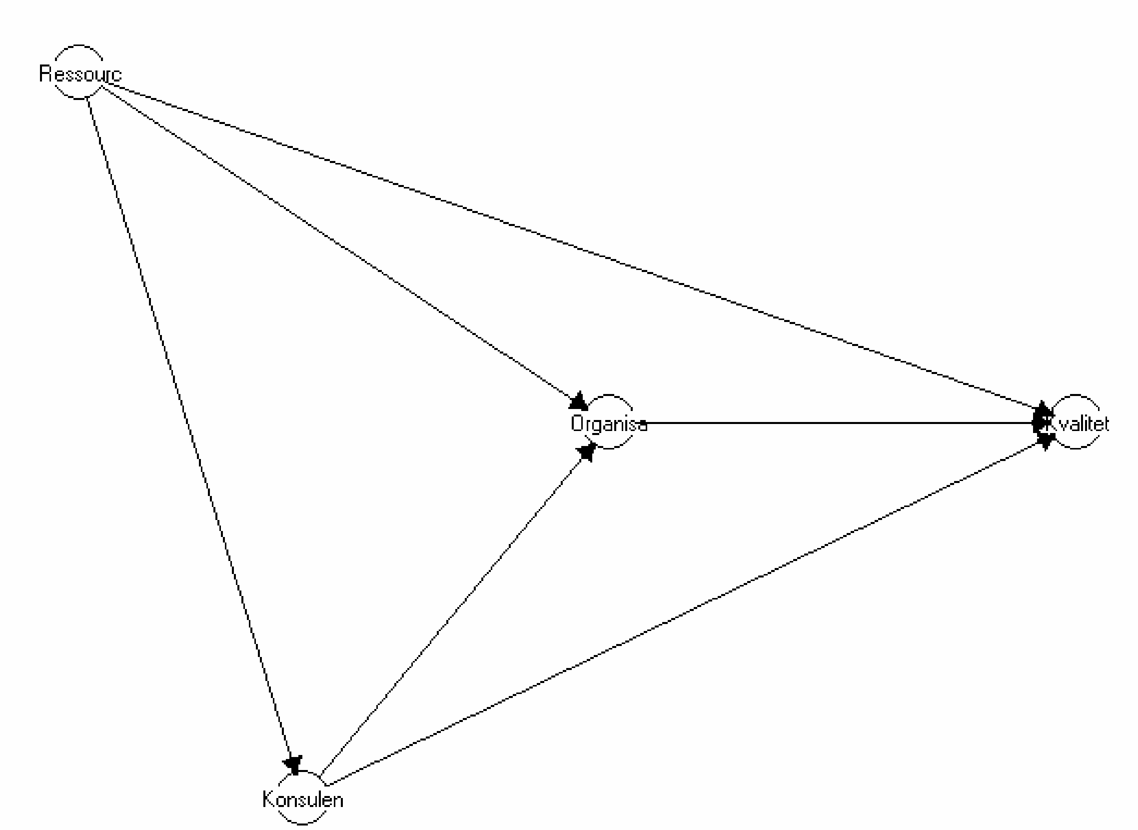
b) The law of conservation of rabbits. If you want to pull a rabbit out of the hat, you have to put a rabbit into the hat.

The Scheines, Glymour and Spirtes research on causality defies the law of conversation of rabbits."
Freedman (1997) (i McKim & Turner (1997))

A study of the quality of home care in Danish municipalities

R = Economic ressources
K = Intervention
O = Organisation of the home care
Q = Quality of hjemmehjælpens

The causal hypothesis: Intervention has a (positive) effect on the organisation and quality of home care.

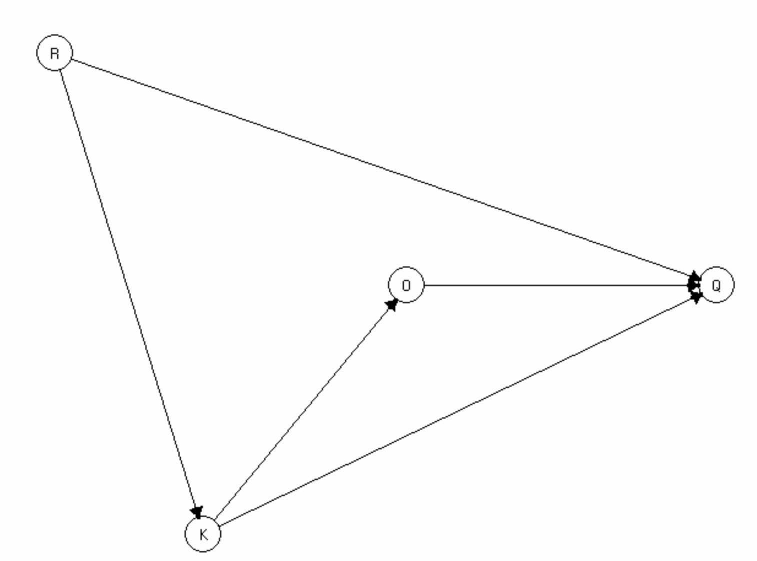


A conventional recursive model (A DAG)

$$P(Q,O,K,R) = P(Q|O,K,R) \cdot P(O|K,R) \cdot P(K|R) \cdot P(R)$$

Different models

No direct effect of R on O



O ⊥ R | K

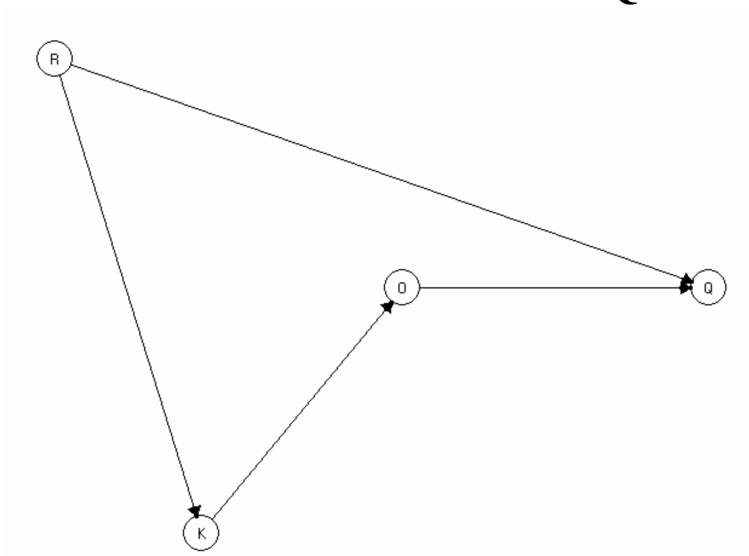
⇔

P(O|K,R) = P(O|K)

⇒

$$P(Q,O,K,R) = P(Q|O,K,R) \cdot P(O|K) \cdot P(K|R) \cdot P(R)$$

No direct effect of K on Q

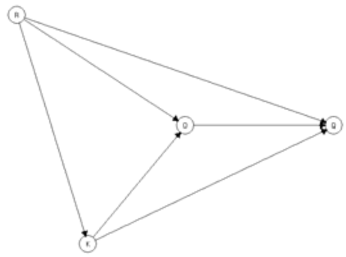


O ⊥ R | K & Q ⊥ K | O,R

⇒

$$P(Q,O,K,R) = P(Q|O,R) \cdot P(O|K) \cdot P(K|R) \cdot P(R)$$

Observational study:



Random intervention:

Replace K with a new variable, K^* such that K^* do not depend on R

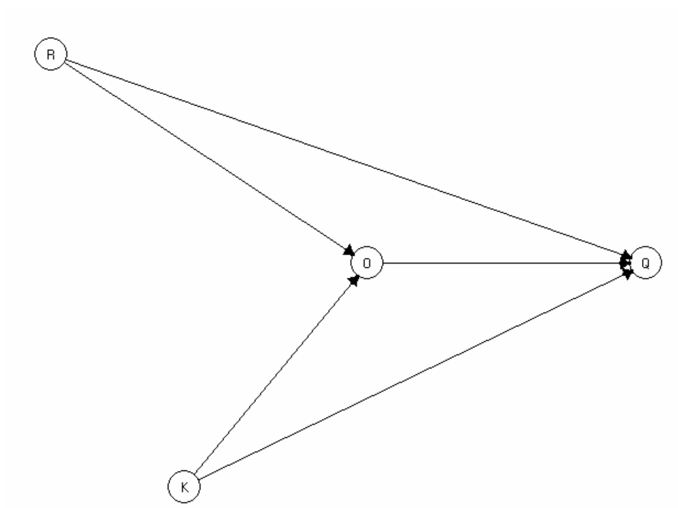
K^* has same outcomes as K

Assume that individual causal effects do not depend on intervention

$$P(O|R,K^*=k) = P(O|R,K=k)$$

$$P(Q|O,R,K^*=k) = P(Q|O,R,K=k)$$

A new graph & a new joint distribution:



$$P^*(Q,O,K,R) = P(Q|O,K,R)\cdot P(O|K,R)\cdot P^*(K)\cdot P(R)$$

Interpretation of results after intervention

1. Evidence of causal effect. If cause and effect are correlated after intervention then the relation must be causal.

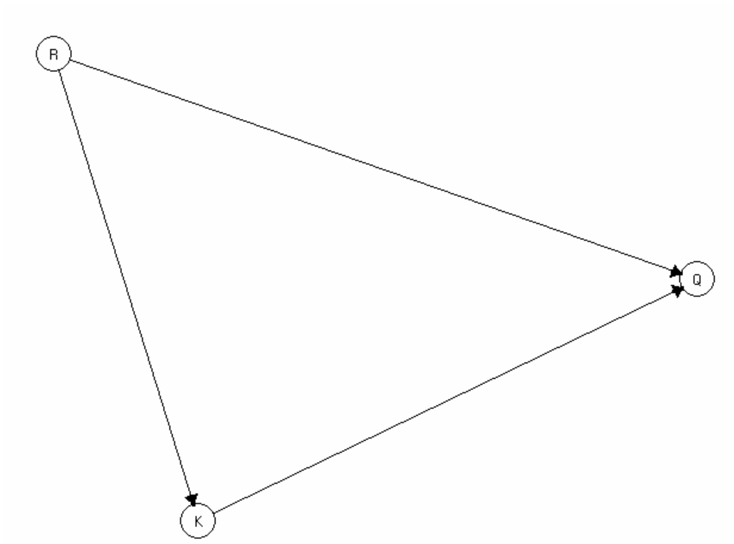
2. $P(Q \mid K^*)$ provides unconfounded estimates of average causal effects.

Interpretation of results in observational studies

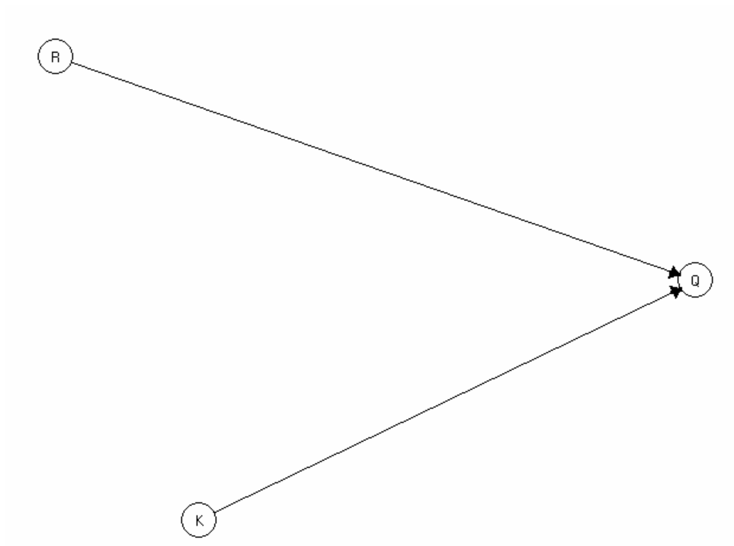
1. No evidence of causality
2. $P(Q|K)$ will in most cases provide confounded estimates of ACE.
3. The estimate of Ace is unconfounded under certain conditions. These conditions may be read off directly from the graph.

No intervening variables

Observational study



Intervention study



The raw ACE is confounded if $P(Q|K) \neq P^{*}(Q|K)$

The Raw ACE

$$P(Q \mid K) = \frac{P(Q,K)}{P(K)}$$

$$= \frac{\sum_R P(Q,K,R)}{P(K)}$$

$$= \frac{\sum_R P(Q \mid K,R) \cdot P(K,R)}{P(K)}$$

$$= \sum_R P(Q \mid K,R) \cdot P(R \mid K)$$

Calculation of ACE in a randomized experiment

$$P^*(Q \mid K) = \frac{P^*(Q,K)}{P^*(K)}$$

$$= \sum_R P(Q \mid K,R) \cdot P^*(R \mid K)$$

$$= \sum_R P(Q \mid K,R) \cdot P(R)$$

- **Two different weighted means unless R and K (observed) are independent**

- **If there is no local (individual) effect of K ($Q \perp K \mid R$) then $P^*(Q|K) = P(Q)$**

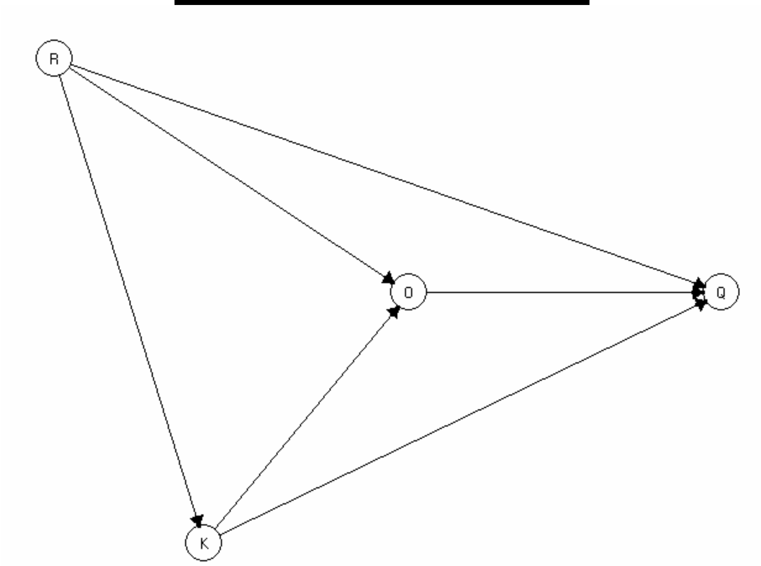
- **$P^*(Q|K)$ depends on the *distribution* of the confounder**

- **$P^*(Q|K) = P(Q|K)$ if $K \perp R$ (no confounding)**

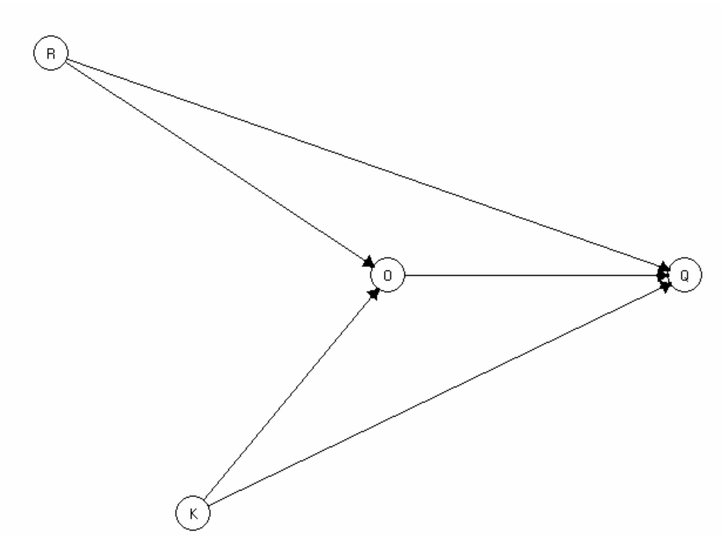
- **$P^*(Q|K)$ may be estimated in an observational study, if confounders have been observed.**

The back-door formula when there are bot spurious and indirect effects

Observational study



Intervention study



$$P(Q \mid K) = \sum_R P(Q \mid K, R) \cdot P(R \mid K)$$

$$P^*(Q \mid K) = \sum_R P(Q \mid K, R) \cdot P^*(R)$$

The intervening variable may be ignored

Proof of the back-door formula:

Observational study

$$P(Q \mid K) = \frac{P(Q,K)}{P(K)}$$

$$= \frac{\sum_{OR} P(Q,O,K,R)}{P(K)}$$

$$= \frac{\sum_{OR} P(Q,O \mid K,R) \cdot P(K,R)}{P(K)}$$

$$= \sum_{OR} P(Q,O \mid K,R) \cdot P(R \mid K)$$

$$= \sum_{R} P(Q \mid K,R) \cdot P(R \mid K)$$

Intervention study

$$P^{*}(Q \mid K) = \sum_{R} P^{*}(Q \mid K,R) \cdot P^{*}(R \mid K)$$

$$= \sum_{R} P^{*}(Q \mid K,R) \cdot P(R)$$

$$P^{*}(Q \mid K,R) = \frac{\sum_{O} P^{*}(Q,O,K,R)}{P^{*}(K,R)} = \frac{\sum_{O} P^{*}(Q,O,K,R)}{P^{*}(K) \cdot P(R)}$$

$$= \frac{\sum_{O} P(Q \mid O,K,R) \cdot P(O \mid K,R) \cdot P^{*}(K) \cdot P(R)}{P^{*}(K) \cdot P(R)}$$

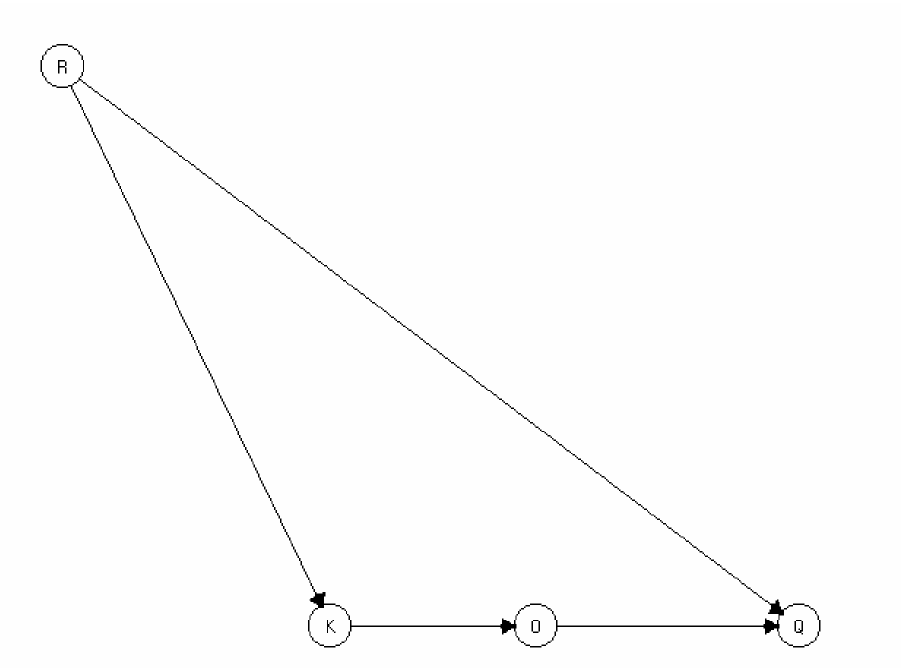
$$= \sum_{O} P(Q,O \mid K,R) = P(Q \mid K,R)$$

$$\Rightarrow$$

$$\mathrm{P}^{*}(\mathrm{Q} \mid \mathrm{K}) = \sum_{\mathrm{R}} \mathrm{P}(\mathrm{Q} \mid \mathrm{K},\mathrm{R}) \cdot \mathrm{P}(\mathrm{R})$$

The front-door formula

No local effect of the confounder on the mediating variable (O)
No direct local effect of intervention (K) on quality (Q)



Under these conditions it follows that ACE does not depend on the distribution of the confounder, but only on P(Q|K,O), P(K) and P(O|K) all of which can be estimated without confounding in an observational study.

Proof of the front-door formula

$$P^*(Q \mid K) = \sum_{OR} P^*(Q,O,R \mid K)$$

$$= \sum_{OR} \frac{P(Q \mid O,R) \cdot P(O \mid K) \cdot P^*(K) \cdot P(R)}{P^*(K)}$$

$$= \sum_{OR} P(Q \mid O,R) \cdot P(O \mid K) \cdot P(R)$$

$$= \sum_{O} \left[P(O \mid K) \sum_{R} (P(Q \mid O,R) \cdot P(R)) \right]$$

We can show that

$$\sum_{R} (P(Q \mid O,R) \cdot P(R) = \sum_{K} P(Q \mid K,O) \cdot P(K) = f(Q,O)$$

such that

$$P^*(Q \mid K) = \sum_{O} \left[P(O \mid K) \cdot f(Q,O) \right]$$

because

$$\sum_{R} (P(Q \mid O,R) \cdot P(R)$$

$$= \sum_{KR} P(Q \mid O,R) \cdot P(K,R)$$

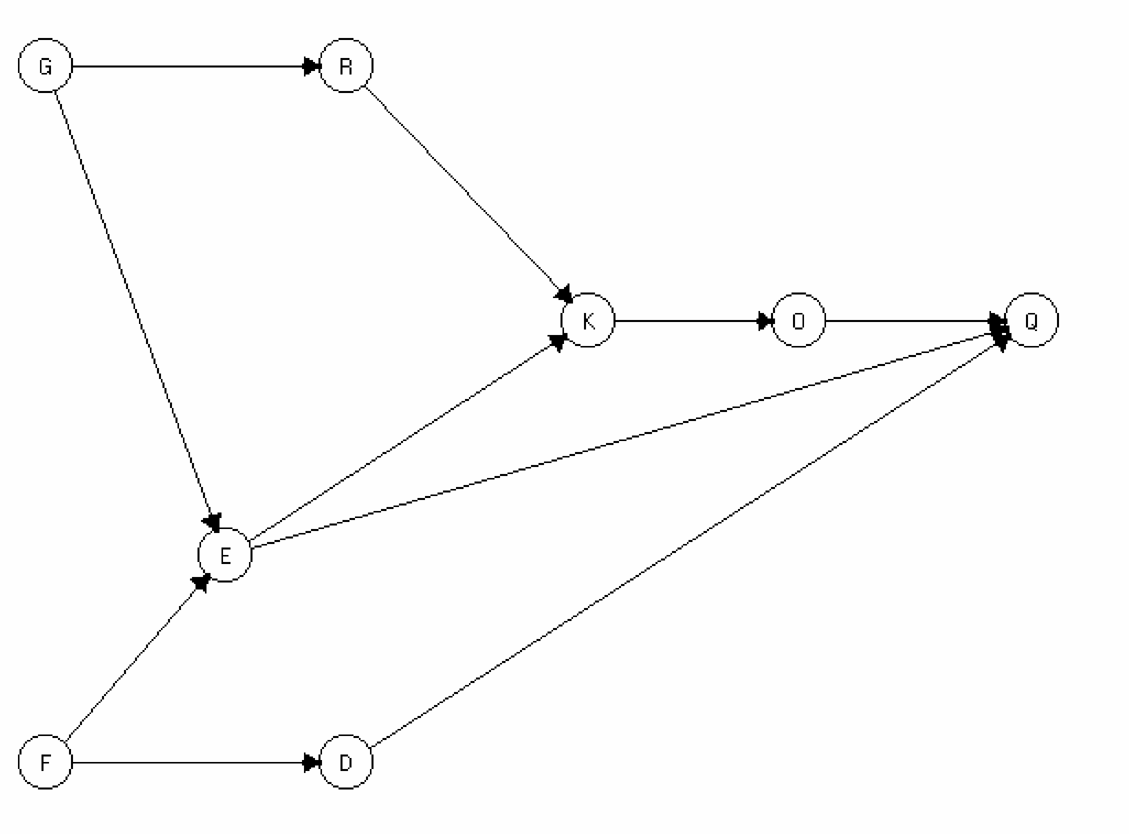
$$= \sum_{KR} (P(Q \mid O,R) \cdot P(R \mid K) \cdot P(K))$$

$$= \sum_{KR} (P(Q \mid O,R,K) \cdot P(R \mid K,O) \cdot P(K))$$

$$= \sum_{KR} (P(Q,R \mid O,K) \cdot P(K))$$

$$= \sum_{K} (P(Q \mid O,K) \cdot P(K))$$

A more complicated realistic example



Which confounders do we need to take into consideration in connection with calculation of the causal effect, $P^*(Q|K)$, in an observational study?

The answer to this question is given by the back-door trick.

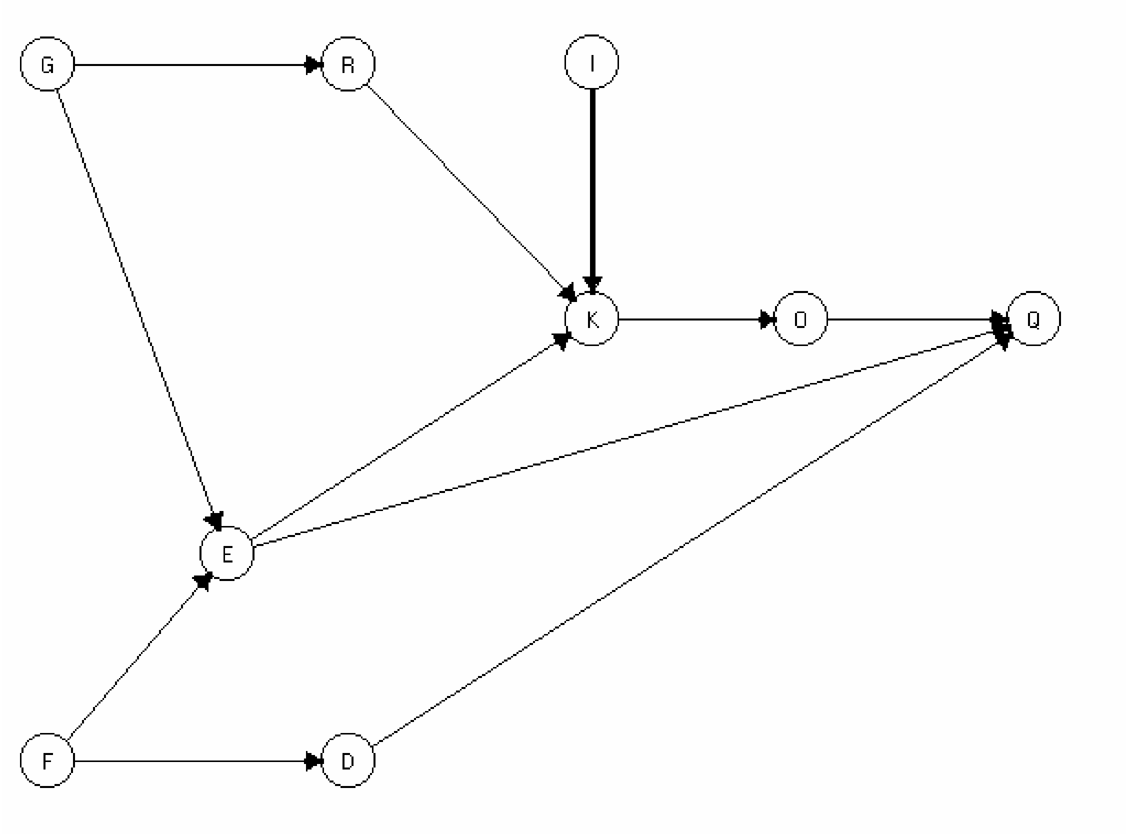
The Back-door Trick

Add a pseudo intervention variable, determining whether or not intervention should be applied, to the graph. The intervention variable must be independent of all variables appearing before the cause (K) in the causal order.

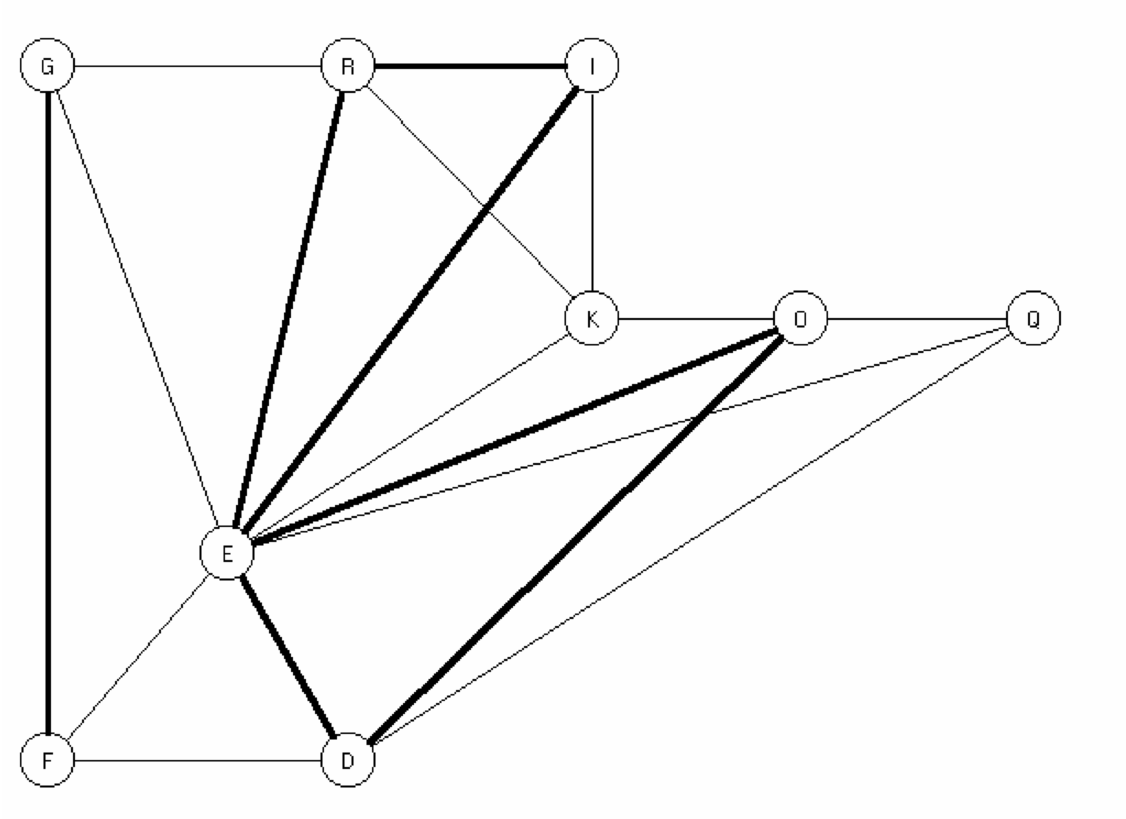
Construct a so-called moral graph marrying all parents to the nodes of the graph.

Find a (minimal) set of variables that *together with the cause* separates the intervention variable from the outcome in the graph. To calculate the causal effect, you only have to take the separators into account.

The intervention graph



The moral graph



Sufficient covariates: E og R eller E og D